Design and realization of high-performance and robust fractional order controllers: An application for automotive systems

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Outline

1. Historical Introduction
2. Background
3. An automotive application
4. Design
5. Tests

G. Maione
FOC for common rail pressure regulation in CNG engines
Leibniz (1646-1716) introduced the symbolic computation $d^n y / dx^n = D^n y$, where $n$ is a non-negative integer.

In a letter to marquis de L'Hôpital (1661-1704), Leibniz raised the following question:

"Can the meaning of derivatives with integer order be generalized to derivatives with non-integer orders?"

L'Hôpital was curious about this and replied by another question to Leibniz: "What if the order will be 1/2?"

Leibniz in a letter dated September 30, 1695 replied: "It will lead to a paradox, from which one day useful consequences will be drawn."
Euler (1707-1783) observed that the result of the evaluation of $d^n y/dx^n$ of the power function $x^p$ has a meaning for non-integer $p$.

Laplace (1749-1827) proposed the idea of differentiation of non-integer order for functions representable by an integral $\int T(t)t^{-x}dt$.

Fourier (1768-1830) suggested the idea of using his integral representation of $f(x)$ to define the derivative for non-integer order.

Abel (1802-1829) wrote very important papers.
Mathematical important contributions: R-L

Riemann (1809-1882), left, contributed for fractional derivative and Liouville (1826-1866), right, for fractional integral of order $\nu \in \mathbb{R}, \nu > 0$

$$aI_t^\nu f(t) = \frac{1}{\Gamma(\nu)} \int_a^t \frac{f(\tau)}{(t-\tau)^{1-\nu}} d\tau \Leftrightarrow \frac{F(s)}{s^\nu}$$

$$aD_t^\nu f(t) = \frac{1}{\Gamma(n-\nu)} \left( \frac{d}{dt} \right)^n \int_a^t \frac{f(\tau)}{(t-\tau)^{\nu-n+1}} d\tau \quad n-1 < \nu < n, \; n \in \mathbb{N}$$

$$\Leftrightarrow s^\nu F(s) - \sum_{k=0}^{n-1} s^k aD_t^{\nu-k-1} f(0)$$

$$\Gamma(x) = \int_0^\infty t^{x-1} e^{-t} dt \quad \Gamma(1) = 1 \quad \Gamma(x+1) = x \Gamma(x), \; x > 0$$
Mathematical important contributions: G-L

Grünwald (1838-1920), left, and Letnikov (1837-1888), right: a formulation useful for time numerical simulation

\[ aD_t^\nu f(t) = \lim_{h \to 0} \frac{1}{h^\nu} \left[ \frac{(t-a)/h}{h} \right] \sum_{i=0}^{[\nu]} (-1)^i \binom{\nu}{i} f(t - i h) \]

where \([x]\) is the integer part of \(x\) and \(\binom{\nu}{i} = \frac{\Gamma(\nu+1)}{\Gamma(i+1)\Gamma(\nu-i+1)}\)

Heaviside (1850-1925):
“There is a universe of mathematics lying in between the complete differentiations and integrations”
More recent mathematical contribution

Caputo (1927-):

\[ aD_t^\nu f(t) = \frac{1}{\Gamma(n-\nu)} \int_a^t \frac{f^{(n)}(\tau)}{(t-\tau)^{\nu-n+1}} d\tau \quad n-1 < \nu < n \]

\[ \Leftrightarrow s^\nu F(s) - \sum_{k=0}^{n-1} s^{\nu-k-1} f^{(k)}(0) \]

With zero in. cond.: \( s^\nu F(s) \)

Other definitions: Ortigueira, the 21st century tool ...
Please give me a break from this weird math!

Do you like this?
Fractals, fractal geometry, self-similarity, chaos, non-linear dynamics... Many examples from nature and physics
Applications: Fractional order systems and dynamics

Many real phenomena, behaviours, and processes are better modelled by differential equations (ODE or PDE) with non-integer order derivatives: fractal phenomena, power laws for long-term memory effects/properties, etc.:

- viscoelasticity in mechanical structures
- biology: diffusion of e.m. waves in tissues, drugs dynamics and diffusion in human body (anesthesia regulation), modelling of pathology in lungs, surface of malignant cell nucleus of breast cancers
- dispersion of pollutants in ocean
- motion of electronic charges in capacitors
- dynamics of stock prices
- behaviour of neurons
- signal processing (ECG, EEG)
- telematics: properties of 3D videos, the “fingerprint”
- anomalous diffusion
Modelling by FODE and FOTF

Linear I-O FODE:

\[
a_n D^{\alpha_n} y(t) + a_{n-1} D^{\alpha_{n-1}} y(t) + \ldots + a_1 D^{\alpha_1} y(t) + a_0 D^{\alpha_0} y(t) = \\
= b_m D^{\beta_m} u(t) + b_{m-1} D^{\beta_{m-1}} u(t) + \ldots + b_1 D^{\beta_1} u(t) + b_0 D^{\beta_0} u(t)
\]

- \(a_i, b_j\): real coefficients
- \(\alpha_i, \beta_j\): arbitrary positive real numbers with \(\alpha_i > \alpha_{i-1}, \beta_j > \beta_{j-1}\)
- \(\alpha_n > \beta_m\)

FOTF:

\[
G(s) = \frac{b_m s^{\beta_m} + b_{m-1} s^{\beta_{m-1}} + \ldots + b_1 s^{\beta_1} + b_0 s^{\beta_0}}{a_n s^{\alpha_n} + a_{n-1} s^{\alpha_{n-1}} + \ldots + a_1 s^{\alpha_1} + a_0 s^{\alpha_0}}
\]

Abel’s famous FODE yields

\[
G(s) = \frac{1}{s^\nu + a}
\]

Today, also \textit{non-linear} fractional order models
Ok, done with some math background

Let's move to control ...
Fractional order control: Ideas by pioneers

Bode (design of feedback amplifiers) (1945): a fractional integrator is the ideal open-loop TF in designing controllers robust to gain/load variations

\[ G(s) = \left( \frac{\omega_{gc}}{s} \right)^{\nu} \]

Tustin (1958): motion control of massive objects by approximating an integrator of order \( \nu = 1.5 \) and nearly constant phase margin of 45° over a wide frequency range around the \( \omega_{gc} \), i.e. between 0.2 \( \omega_{gc} \) and 1.4 \( \omega_{gc} \)

Manabe (1961): dynamics of flexible spacecraft structures
Fractional order control: Recent famous achievements

CRONE control, Bordeaux (FR) team head by Oustaloup (1980): car suspensions control, control of industrial/thermal plants, etc.

TID controller (1994): Lurie, Three-parameter tunable tilt-integral-derivative (TID) controller, patent US5371670 for systematic setting of a fractional integrator plus I & D actions

$\text{PI}^\lambda \text{D}^\mu$ controllers (1999): Podlubny, *IEEE TAC*, 44 (1), pp. 208-214: $G_c(s) = K_P + \frac{K_I}{s^\lambda} + K_D s^\mu \rightarrow$ A generalization


Monje, Vinagre, Feliu, Chen (2008): “Tuning and auto-tuning of fractional order controllers for industry applications”. *Control Engineering Practice*, 16, pp. 798-812

Literature (control, robotics, system identification, etc.): non-integer order controllers, FOPID contr., FO(PI) contr., FOC, PI$^\alpha$, (PI)$^\alpha$ ...
Fractional order controller: Basic idea (1)

$s^\nu$, with $\nu \in \mathbb{R}$, allow to obtain slopes that are fractions of the usual $\pm 20 \text{ dB/decade}$ and phase shifts that are fractions of $\pm \pi/2$. 

\[
G(s) = \left(\frac{\omega_{gc}}{s}\right)^\nu
\]

$\omega_{gc} = 1 \text{ rad/s}$
The open-loop frequency response is shaped (mag. and phase diagram) to meet performance and robustness specifications and to obtain a “flat” phase diagram in a sufficiently wide range around the crossover frequency. Flatness implies higher robustness.
Fractional order controllers: Benefits and Issues

- **Improvement** of closed-loop performance and robustness, if the plant is an integer order system and especially if it is modelled as a non-integer order system.
- **Impact on industrial control loops** and on digital applications based on PID.
- **Flexibility**: more design degrees of freedom, i.e. $\lambda$ and $\mu$.

- A rational approximation with minimum-phase zeros and stable poles is required for implementation.
- Quality of analog/discrete approximation depends on location of zeros and poles (usually interlaced).
Have a deep breath...
CNG engines: Motivation and issue

Why Compressed Natural Gas (CNG) engines? 
*To reduce consumptions and pollution and maintain or increase performance*

**Pressure control** to accurately meter the air-fuel mixture that strongly affects combustion efficiency

Difficulty: gas compressibility make the working conditions vary to a large extent
The injection system: Components and operation (1)

A fuel tank of high pressure gas
A mechanical pressure reducer (PR): a main chamber (MC) & a control chamber (CC)
A fuel metering system: common rail and 4 electro-injectors
A solenoid valve (SV) regulates the flow into CC
An Electronic Control Unit (ECU)

The injection flow only depends on:
- the $p_{rail} \approx p_{MC}$
- the injectors opening time intervals $t_j$ driven by ECU

Tank: 30-200 bar ⇒ Common rail: 5-25 bar
The injection system: Components and operation (2)

ECU determines $x_2 = p_{\text{rail}}$ and controls gas flow:

1. sets injection timings depending on engine speed and load
2. PWM of the duty cycle of the driving current to open/close the SV $\Rightarrow$ pressure in CC: $p_{\text{CC}}$

Inflow section of the PR is varied by displacing a shutter (S) coupled with a piston (P)

Gas in MC pushes P up & gas in CC pushes P down

1. If SV is energized, gas enters CC and pushes P down: S is opened, more fuel enters MC, then $p_{\text{MC}}$ increases
2. If SV is not energized, $p_{\text{CC}}$ decreases, P raises and S is closed by action of a preloaded spring
The injection system: Non-linear model

Assumptions:
1. the control chamber and the rail circuit have constant temperature and uniform pressures: $x_1 = p_{CC}$, $x_2 = p_{rail}$
2. Tank pressure $p_{tk}$: measure always available (fuel supply) and $\approx \text{const}$ in a large time interval
3. $u_1, u_2$: commands to valve and injectors
4. Injection pressure $p_{rail} \Rightarrow$ no modeling of electro-injectors

\[
\begin{align*}
\dot{x}_1(t) &= c_{11} p_{tk}(t) u_1(t) - c_{12} \sqrt{x_2(t)} [x_1(t) - x_2(t)] \\
\dot{x}_2(t) &= c_{21} p_{tk}(t) [c_{24} x_1(t) - c_{25} x_2(t) - c_{26} p_{tk}(t) - c_{27}] - c_{22} x_2(t) u_2(t) \\
&\quad + c_{23} \sqrt{x_2(t)} [x_1(t) - x_2(t)]
\end{align*}
\]

Model describes the complex dynamics in several equilibrium points

Main controlled variable for CNG injection: $x_2$
**The injection system: Linearization**

**Different equilibrium working points** ⇒ **Different tunings of the controller**

*Choice that works because control must keep the pressure close to a reference value, depending on driver power request, engine speed and load*

\[
\begin{align*}
\delta\ddot{x}(t) &= A\delta x(t) + B\delta u(t) \\
\delta y(t) &= \delta x_2(t) = C\delta x(t)
\end{align*}
\]

\[
\begin{align*}
\delta x(t) &= x(t) - \bar{x}, \delta u(t) &= u(t) - \bar{u}
\end{align*}
\]

The Laplace transform of the linearized model yields:

- \( p_{tk} \) greatly affects the control action through \( G_{12} \)
- Injection process as a disturbance on the rail pressure
- Control chamber and rail pressure dynamics are strongly coupled
The injection system: Equilibrium point

Working point: to inject the proper fuel amount (depending on power request, speed and load), ECU sets $p_{rail}$ and $t_j$ by look-up tables

- operation strongly depends on $\bar{p}_{tk}, \approx \text{const}$ during injection
- the injection flow rate only depends on $p_{rail} = x_2$

Optimal injection profile is set for each working point, that refers to specific values of $p_{tk}, p_{rail}$, and injection timings $t_j$

- **INJECTORS**: $\bar{u}_2 = 4 \cdot t_j \cdot \nu / 120$ is the mean of injection frequency
- **VALVE**: $\bar{u}_1$ is the mean duty cycle of driving current
- $\bar{p}_{tk}, \bar{x}_2$ and $\bar{u}_2$ are set $\Rightarrow \bar{u}_1$ and $\bar{x}_1$ computed from the nonlinear model
The injection system: Linearized model

We consider only $\delta \bar{u}_1$, and indicate it as $u$. Then:

$$y(s) = \frac{a_{21} b_{11}}{s^2 - (a_{11} + a_{22}) s + a_{11} a_{22} - a_{12} a_{21}} u(s) \approx \frac{K}{1 + T s} e^{-\tau s} u(s)$$

$\tau$ is for the pressure propagation from the main chamber to the common rail

$K = a_{21} b_{11}/(s_1 s_2)$, $T = -1/s_1$, $\tau$ is experimentally determined

$\Rightarrow$ Sufficient accuracy to represent the nonlinear CNG injection system
(tests by simulation in different working points)

$\Rightarrow$ Comparison of different controllers designed for FOPTD systems
The Control Strategy

Several equilibrium points: $K$, $T$, $\tau$ change $\Rightarrow$ Gain scheduling to adapt the controller to variations

For each $p_{tk}$, the working point is established by the $p_{rail}$ and by the average duration of injection. For each point, a different FOC:

a. consider the variation from current to new required point
b. design FOC with reference to the new point

Ok if variations < 2 bar for rail pressure and 6 s for injection duration

If variations are higher, then consider several intermediate values

Note 1: stability in switching is tested by detailed simulation
Local stability by FOC and steps to intermediate values are bounded

Note 2: it is usual to employ maps of PI-gains (tuning by Z-N) for the working points
Have another breath...
Robust stability by $D$-decomposition

Applied to integer order systems to design & determine all stabilizing controllers

Aim: set of controller gains leading to closed-loop stability

Why?

- Changes in working conditions in injection: knowing the set of the gains avoids time-consuming stability checks for any new controller settings and makes the tuning faster

- If the chosen controller gains correspond to a point of the set that is far from its boundary, then stability is still guaranteed for bounded variations

A stability domain: remember Nyquist?
Stability regions by $D$-decomposition

Open-loop TF: $G(s) = \frac{K_I (1 + T_I s^\nu)}{s^\nu} \frac{K e^{-\tau s}}{(1 + T s)}$

Closed-loop TF: $F(s) = \frac{K K_I (1 + T_I s^\nu) e^{-\tau s}}{(1 + T s) s^\nu + K K_I (1 + T_I s^\nu) e^{-\tau s}}$

Fractional order characteristic pseudo-polynomial equation:

$E(s) = (1 + T s) s^\nu + K K_I (1 + T_I s^\nu) e^{-\tau s} = 0$

Domain $\mathcal{D}$ of all stabilizing controllers: in the parameter space associated to the triple $(K_P, K_I, \nu)$, such that all roots are LHP $\mathcal{D}$ is defined by a real root boundary (RRB), an infinite root boundary (IRB), and a complex root boundary (CRB)
Stability regions by $D$-decomposition: Boundaries

IRB: for $s \to \infty$, not existing

RRB from $E(s = 0) = 0$: $K_I = 0$

CRB from $E(s = j\omega) = 0$:

\[
K_I(\omega) = \frac{\omega^\nu (\sin(x) + \omega T \cos(x))}{K S}
\]

\[
K_P(\omega) = \frac{(\omega TS - C) \sin(x) - (S + \omega TC) \cos(x)}{K S}
\]

$\theta = \frac{\pi}{2} \nu, x = \omega \tau$

$C = \cos(\theta), S = \sin(\theta)$
If we fix $\nu$, then a curve in a 2D-space $(K_P, K_I)$ for $\omega$ from 0 to $\infty$. 

Stability regions by $D$-decomposition
Relative stability lines by $D$-decomposition

PM line: increase the dead-time in $G(s)$ by $PM_s$: $x \rightarrow y = x + PM_s$

GM line: amplify $G(s)$ by $GM_s$: $K \rightarrow K GM_s$

- **Point $x$:** $PM_s$, $GM_s$, crossover frequencies $\omega_{gc}$ and $\omega_{pc}$
- Distance between the CRB curves and design points (relative stability lines) measures the robustness level
Did I break your synapses?
Design by loop-shaping: Starting formulas

\[ G_p(s) = \frac{Ke^{-\tau s}}{1 + Ts} \]

\[ G_c(s) = K_P + \frac{K_I}{s^\nu} = \frac{K_I}{s^\nu} (1 + T_I s^\nu) \quad T_I = K_P / K_I \quad 1 < \nu < 2 \]

\[ G(j\omega) = G_c(j\omega)G(j\omega) = \frac{KK_I}{\omega^\nu} \left\{ \cos(0.5 \nu \pi) + j\sin(0.5 \nu \pi) \right\} \left(1 + j\omega T_I\right) \]

If we use \( u = \omega T \):

\[ G(ju) = \frac{KK_I T^\nu}{u^\nu} \left\{ 1 + T_I \left( \frac{u}{T} \right)^\nu \cos(0.5 \nu \pi) + j\sin(0.5 \nu \pi) \right\} \left(1 + ju\right) e^{-j\frac{u\tau}{T}} \]
Design by loop-shaping: Main idea
Approximation of perfect I-O tracking and fractal robustness

\[ F(ju) = \frac{1}{1 + G^{-1}(ju)} \]

\[ |F(ju)| \approx 1 \iff y(ju) \approx r(ju) \text{ in a limited bandwidth } u_B = \omega_B T_E \]

For a stable performance despite changes in parameters, shape the open-loop freq. resp. around the gain crossover by using the fractional I to obtain \( PM_s \), held nearly constant in a wide range around the crossover \( u_C \) (a flat phase diagram and a magnitude diagram with fractional slope of \(-20 \nu \) dB/decade in this range) \( \Rightarrow \) Selection of \( T_I \) and \( \nu \)
Design method: Enforcing the specifications

1st requirement: \( u_B \) ensuring a good tracking response

Trade-off between fast closed-loop response and placement of \( u_C \) in a central position in the flat region of the phase diagram

Estimation \( u_C \in [\frac{u_B}{1.7}, \frac{u_B}{1.3}] \): \( u_C = \frac{u_B}{1.5} \)

2nd requirement: a phase margin \( PM_s \) in a wide range around \( u_C \)

\[
PM = \tan^{-1} \left( \frac{T_I \left( \frac{u_C}{T} \right)^{\nu} \sin(0.5 \nu \pi)}{1 + T_I \left( \frac{u_C}{T} \right)^{\nu} \cos(0.5 \nu \pi)} \right) - \tan^{-1}(u_C) - \frac{u_C \tau}{T} + \pi - \frac{\pi}{2} \nu
\]

\[
= \varphi_1(u_C) - \varphi_2(u_C) - \frac{u_C \tau}{T} + \pi - \frac{\pi}{2} \nu
\]

Select \( T_I \) s.t. \( \varphi_1(u_C) - \varphi_2(u_C) - \frac{u_C \tau}{T} = 0 \). Then: \( PM = (2 - \nu) \frac{\pi}{2} \)
Design by L-S: Relation between $\nu$ and phase margin

$$PM = (2 - \nu_s) \frac{\pi}{2} \quad \nu = 2 - \frac{PM_s}{\pi/2}$$

A direct relation between the fractional order and the specified phase margin!

<table>
<thead>
<tr>
<th>$\nu$</th>
<th>$PM$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3</td>
<td>63°</td>
</tr>
<tr>
<td>0.4</td>
<td>54°</td>
</tr>
<tr>
<td>0.5</td>
<td>45°</td>
</tr>
<tr>
<td>0.6</td>
<td>36°</td>
</tr>
</tbody>
</table>
Design by L-S: Resolving formula for $T_I$

$$T_I = \frac{u_C + \rho}{(u_C T)^\nu} \frac{1}{(S - u_C C - \rho (C + u_C S))}$$

$$\rho = \tan \left( \frac{u_C \tau}{T} \right)$$

$$S = \sin(0.5 \nu \pi)$$

$$C = \cos(0.5 \nu \pi)$$
Design by L-S: Resolving formula for $K_I$

Gain crossover: $|G^{-1}(j u_C)|^2 = 1$

$$\frac{1}{K^2 K_I^2} \left( \frac{u_C}{T} \right)^{2\nu} \frac{1 + u_C^2}{1 + 2 T_I \left( \frac{u_C}{T} \right)^\nu C + T_I^2 \left( \frac{u_C}{T} \right)^{2\nu}} = 1$$

$C = \cos(0.5 \nu \pi)$

$$\bar{K}_I = \frac{1}{K} \left( \frac{u_C}{T} \right)^\nu \sqrt{\frac{1 + u_C^2}{1 + 2 T_I \left( \frac{u_C}{T} \right)^\nu C + T_I^2 \left( \frac{u_C}{T} \right)^{2\nu}}}$$

G. Maione  
FOC for common rail pressure regulation in CNG engines
Algorithm of design procedure

1. Fix $\nu$. Specifications: $u_B$, $PM_s$. Then: $u_C = \frac{u_B}{1.5}$ and $\nu = 2 - \frac{PM_s}{\pi/2}$

2. Set $T_I$: $T_I = \overline{T}_I = \frac{u_C + \rho}{(\frac{u_C}{T})^\nu (S - u_C C - \rho (C + u_C S))}$

3. Set $K_I$: $K_I = \overline{K}_I = \frac{1}{K} \left( \frac{u_C}{T} \right)^\nu \sqrt{\frac{1 + u_C^2}{1 + 2 \overline{T}_I \left( \frac{u_C}{T} \right)^\nu C + \overline{T}_I^2 \left( \frac{u_C}{T} \right)^{2\nu}}}$

   Determine $\overline{K}_p = \overline{T}_I \overline{K}_I$

4. Approximate $s^\nu$ then $G_c(s) = \overline{K}_p + \frac{\overline{K}_I}{s^\nu}$
An approximation by a truncated CFE

A rational approximation with minimum-phase zeros and stable poles is required for implementation. Efficient approximation of $s^\nu$, for $0 < \nu < 1$, from truncation of a CFE (Maione, 2008):

$$s^\nu \approx \frac{\alpha_{N,0}(\nu) s^N + \alpha_{N,1}(\nu) s^{N-1} + \ldots + \alpha_{N,N}(\nu)}{\beta_{N,0}(\nu) s^N + \beta_{N,1}(\nu) s^{N-1} + \ldots + \beta_{N,N}(\nu)}$$

$$\alpha_{N,j}(\nu) = \beta_{N,N-j}(\nu) = (-1)^j \binom{N}{j} (\nu + j + 1)(\nu - N)_j$$

$$(\nu + j + 1)(\nu - N)_j = (\nu + j + 1)(\nu + j + 2) \cdots (\nu + N)$$

$$(\nu - N)_j = (\nu - N)(\nu - N + 1) \cdots (\nu - N + j - 1)$$

⇒ Closed-form formulas

⇒ Interlaced negative real $z$ & $p$ is guaranteed

If $s^\nu$, with $1 < \nu < 2$, then $s^\nu = s s^{\nu-1}$ and approximate $s^{\nu-1}$
Approximation with only $N = 4$ z-p pairs

Fractional differentiator $s^{0.5}$
Approximation by CF (Maione)

Angular frequency [rad/s]
Phase [degrees]
Need some air?
Simulation tests

- **Non-linear state-space model** implemented in the Matlab/Simulink environment
- AMESim package, a multidomain virtual prototyping tool
- Each working condition of the CNG injection system provides a different triple \((K, T, \tau)\)
- Each \((K, T, \tau)\) has an associated FOPI controller

\[
uB = 8.55 \text{ then } \omegaB = \frac{uB}{T}\]

Some good values of the fractional order \(\nu: 0.3, 0.4, 0.5, 0.6\)

\(N = 5\) zero-pole pairs to approximate the fractional integrator

*Important:* to limit overshoot/steady-state errors/settling times in \(p_{\text{rail}}\) because oversupply/undersupply of fuel alters the air-fuel ratio and increases consumptions and emissions
First test: Small step in the reference pressure

Injectors exciting time interval: 5 ms
Engine speed: 2500 rpm
Tank pressure: 50 bar
Only one PI (with open-loop Z-N rules) or FOPI

Increase in o.s. then inaccurate metering of the injected fuel by PI
Problems much more with PI:
- disturbances and nonlinearities related to injectors operations
- PWM modulation of the solenoid valve command
- saturation
Second test: Large step in reference pressure

3 PIs or FOPIs are used
Third test: Disturbance rejection

Tank pressure: 50 bar  
Rail pressure reference: 5 bar

Variation at $t = 2.5\text{ s}$: engine speed $2500 \rightarrow 5000\text{ rpm}$ then injectors exciting time interval $3 \rightarrow 8\text{ ms}$

The injected fuel increases but initially the valve can not maintain the reference rail pressure that decreases:

- FOPI promptly reacts
- PI has much more overshoot, then more problems in fuel regulation

FOPI compared to a Generalized Predictive Controller:

- FOPI yields much lower rise time and similar settling time
- FOPI has inherent fractal robustness
- FOPI is less complex
Conclusions

- FOPI control of rail pressure for injection in CNG engines
- **Loop-shaping to design FOPI:**
  - Optimality of feedback system
  - High robustness to changes in gain & internal parameters
  - Good closed-loop performance
  - Ability to reject load disturbances
- **Closed-form formulas** to relate performance & robustness specifications to \((K_P, K_I, \nu)\) of the FOPI controller
- **Gain scheduling** for adapting the controller to different working conditions
- Test by commercial sw & experiments
In Italy, the shortest path between two points is an arabesque *(Flaiano, famous Italian writer)*

In Bank of Italy, for the shortest path you need fractal geometry *(Unknown)*

What about career in academics?
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